GCSE Maths – Algebra

Equation of a Circle and its Tangent

(Higher Only)

Worksheet

NOTES

SOLUTIONS

This worksheet will show you how to work out different types of questions involving circles and their tangents. Each section contains a worked example, a question with hints and then questions for you to work through on your own.

Section A

Worked Example

A circle centred at the origin has a radius of 4. What is the equation of this circle?

Step 1: Use the basic equation of a circle centred at the origin $x^2 + y^2 = r^2$.

We know that the radius is 4, so we can square this to get the value of r^2.

$$r^2 = (4)^2 = 16$$

Therefore, the equation of this circle is:

$$\boldsymbol{x^2 + y^2 = 16}$$

Guided Example

A circle centred at the origin has the equation $x^2 + y^2 = 89$.

What is the radius of this circle?

Step 1: Use the general equation of a circle centred at the origin $x^2 + y^2 = r^2$.

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

1. What is the equation of a circle centred at the origin with radius of 9?

2. What is the radius of a circle that has the equation $x^2 + y^2 = 56$?

3. A circle passes through the points $(8, 0)$, $(0, -8)$, $(-8,0)$ and $(0, 8)$. What is the equation of this circle?

4. A circle passes through the point $(3, 7)$ with its centre at the origin $(0, 0)$. What is the equation of this circle?

Section B

Worked Example

A circle has the equation $x^2 + y^2 = 98$.
What is the equation of the tangent that touches the circle at $(7, 7)$?

Step 1: Work out the gradient of the radius from the origin $(0, 0)$ to the point that touches the tangent.

To work out the gradient of the radius, we need to calculate the difference in y-coordinates divided by the difference in x-coordinates:

$$Gradient\ of\ radius\ connecting\ (0,0)\ to\ (7,7) = m_r = \frac{7-0}{7-0} = \frac{7}{7} = 1$$

Step 2: Take the negative reciprocal of the radius gradient to find the gradient of the tangent.

$$Gradient\ of\ tangent = m = -\frac{1}{Gradient\ of\ radius} = -\frac{1}{m_r} = -\frac{1}{1} = -1$$

Step 3: Calculate the equation of the tangent by substituting in the values of x and y that the tangent is known to pass through.

A straight line has the general form $y = mx + c$. For the tangent we know $m = -1$, so the tangent has equation $y = -x + c$. The tangent passes through the point $(7, 7)$, so use these values in to obtain the value of c:

$$7 = -1(7) + c$$
$$7 = -7 + c$$
$$c = 14$$

The equation of the tangent is $\boldsymbol{y = -x + 14}$.

Guided Example

A circle has the equation $x^2 + y^2 = 40$. Find the equation of the tangent that touches the circle at $(-6, -2)$.

Step 1: Work out the gradient of the radius from the origin (0, 0) to the point that touches the tangent.

Step 2: Take the negative reciprocal of the radius gradient to find the gradient of the tangent.

Step 3: Calculate the equation of the tangent by substituting in the values of *x* and *y* that the tangent is known to pass through.

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

5. A circle has the equation $x^2 + y^2 = 85$. Find the equation of the tangent that touches the circle at $(-2, 9)$.

6. A circle has equation $\frac{x^2+y^2}{2} = 40$. Find the equation of the tangent that touches the circle at $(8, 4)$.

7. A circle has the equation $x^2 + y^2 = \frac{37}{2}$. Calculate the gradient of the tangent that touches the circle at $\left(\frac{5}{2}, \frac{7}{2}\right)$.